New algorithm for the source field component determination with resistive sheet and coaxial type conductors in $T-\Omega$ formulation

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In $T-\Omega$ formulation, since the tree-cotree based calculation of source field component is an automatic and random procedure, some additional challenges arise. One case is the calculation of the source field component on the contour of the coaxial conductor terminal. Another case is the calculation of the source field component in the source conductor with resistive sheet. In this paper, new algorithm for source field component calculation is developed to handle the above two problems.

***Index Terms*—Finite element analysis, Electromagnetic simulation, Source field component, $T-\Omega$ formulation.**

I. Introduction

In low frequency electromagnetic simulation, finite element method is a very efficient algorithm. It has two advantages: 1) it is accurate in describing the complicated geometry which is the case in most of the low frequency applications. 2) it is flexible at handling inhomogeneous materials with nonlinearity that is common in low frequency applications. Among different FEM formulation, $T-\Omega$ formulation is able to provide high order accuracy with small number of unknowns. In $T-\Omega$ formulation, source field component due to current excitations can be represented by edge elements and denoted by H_p in this paper. H_p needs to be precalculated on each edge of the mesh in the whole region before the simulation. H_p should satisfy the criteria $\nabla\times H_p = J$ with J as the source current density.

The calculation of H_p becomes challenging when the conducting region is not simply-connected region. Special treatments are introduced to solve such problems [1]-[5]. H_p calcluation is further challenged in the following two cases: 1) there is resistive sheet in the source conductors. 2) the source terminal is coaxial type. In this paper, new algorithm has been developed to overcome these two obstacles. Two examples are used to demonstrate the effectiveness of the proposed algorithm.

II. Basic concept of H_p determination

In $T-\Omega$ formulation, the solution process consists of two steps. The first step is to calculate H_p associated with prescribed current excitations. The second step is to solve the entire system equations based on pre-calculated H_p. In the above formula, H_p is defined on edges. H_p calculation can be implemented in different ways. Following [2] in this paper, H_p is calculated in the following four steps:

1) Assign H_p on the contour of the conductor terminal and ascertain that the integration of H_p along the contour equals the total current flowing through the terminal.

2) Assign H_p on the surface of the source conductor by ascertaining that the integration of the H_p along all edges of each surface triangle is zero since there is no current flowing out of the conductor surface.

3) Assign H_p in the non-conductor region by ascertaining that the integration of the H_p along all edges of each triangle in non-conductor region is zero since there is no current flowing in the non-conducting region. When the source conductor is not simply-connected, special treatment is needed in step 2) and 3) to generate a cutting domain [4].

4) Calculate H_p inside the source conductor based on the following Galerkin equation where R_s represents the source conductor region, t_i is in the space of edge elements.

$$\int_{R_s} \nabla\times t_i \cdot \frac{1}{\sigma} \cdot \nabla\times H_p = 0 \quad (1)$$

H_p values on all edges of all conductor boundaries have already been determined. The above equation is used to determine the H_p inside the source conductor.

III. Resistive sheet handling

When a problem includes a very thin conducting layer, it can be modeled either by a very thin 3D geometry or be modeled by a resistive sheet. Modeling the thin 3D geometry may result in considerable large amount of mesh elements with possible bad quality. This study attempts to directly work on the resistive sheet with zero thickness. For the resistive sheet, resistance R is given. The current density is supposed to be perpendicular to the resistive sheet. To this end, (1) need to consider the impact of the resistive sheet.

Suppose the area of the sheet is S. The thickness of the sheet is δ. The conductivity of the sheet is σ. The resistance of the sheet is $R=\frac{\delta}{\sigma S}$. The lhs of (1) on the resistive sheet can be derived as

$$\begin{aligned}\lim_{\delta\to 0}\int_{V_R} \nabla\times t_i \cdot \frac{1}{\sigma} \cdot \nabla\times H_p dV &= \lim_{\delta\to 0}\int_{S_R} \nabla_t\times t_i \cdot \frac{1}{\sigma} \cdot \nabla_t\times H_p \delta dS \\ &= \lim_{\delta\to 0}\int_{S_R} \nabla_t\times t_i \cdot \frac{RS}{\delta} \cdot \nabla_t\times H_p \delta dS \\ &= \int_{S_R} \nabla_t\times t_i \cdot (RS) \cdot \nabla_t\times H_p dS \quad (2)\end{aligned}$$

where S_R represents the resistive sheet and V_R represents the thin conducting layer.

The above formula is further added to (1) as

$$\int_{R_s} \nabla\times H_p \frac{1}{\sigma} \nabla\times T + \int_{S_R} \nabla_t\times H_p \cdot (RS) \cdot \nabla_t\times T dS = 0 \quad (3)$$

The loss on the resistive sheet can be derived by

$$\lim_{\delta\to 0}\int_{V_R} \frac{J^2}{\sigma} dV = \lim_{\delta\to 0}\int_{S_R} \frac{J^2 RS}{\delta} \delta dS = \int_{S_R} J^2 (RS) dS \quad (4)$$

IV. Coaxial conductors handling

The handling of a coaxial conductor in $T - \Omega$ formulation is not trivial. As mentioned above, when the H_p is assigned on the contour of the terminal, the integration of the H_p along the terminal contour should equal the total current flowing through the terminal. This is enforced by the following scheme. All the edges is assigned to zero H_p except for one edge whose H_p is determined such that the integration of H_p on that edge equals the total prescribed current over the terminal. This process is a random procedure, that is, the edge with nonzero H_p is randomly chosen.

This process is always valid as long as the conductor associated with the terminal is solid. But for the terminal with a hole, it becomes problematic. Considering a conductor terminal with a hole in the middle of it, there are two separate contours: the outer contour enclosing the terminal and the inner contour enclosing the hole. Physically, nonzero H_p can only be assigned to the outer contour since the integration of the H_p should be zero on the inner contour due to the fact that there is no current flowing through the hole. But the H_p is assigned randomly on the contours, so nonzero H_p can be either on the inner contour or on the outer contour, which leads to additional difficulty for coaxial conductors. For practical applications, different conductors normally bundled as a coaxial cable have an integral effect on each other. On the terminal of outer conductors, it is possible that nonzero H_p exists on both inner contour and outer contour. Obviouly this H_p assignment can not be handled by random procedure mentioned above.

In this study, the new algorithm is developed for the correct H_p assignment on the terminals of coaxial conductors automatically. The basic idea of the algorithm is to assign the value of the nonzero H_p on the inner contour of the innermost conductor at first, then assign the nonzero H_p on the other contours of the terminals one by one in an inside-out order. More details will be discussed in the full paper.

V. Application examples

The first application is used to examine effectiveness of the resistive sheet. In this example, there is an 8-shaped conductor with 1mm thickness as shown in Fig. 1.

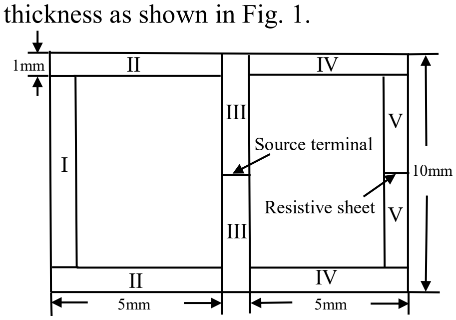
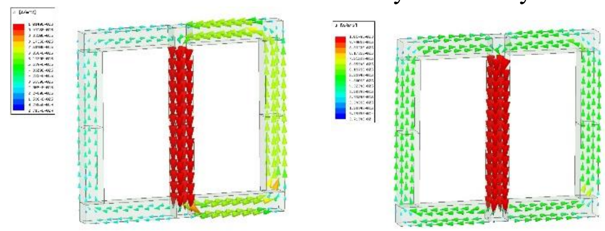


Fig. 1. The shape and the size of the conductor

The total current (1A) flowing through the source terminal in the middle bar is divided into the left loop (region I, II, III) and the right loop (region III, IV, V). The conductivity of the region I, II, III, IV is 1e7 Siemens/m. The conductivity of the region V is 1e9 Siemens/m. Thus, the resistance of the region I, 8e-4 Ohm, is much larger than the resistance of the region V, 8e-6 Ohm. As a result, the current flowing to the left loop should be much smaller than the right loop. If add the resistive sheet with resistance 7.92e-4 (= 8e-4 – 8e-6) Ohm, the effective resistance of the left branch and the right branch becomes the same. The current flows to the left branch and right branch should also be the same. This can be demonstrated by the simulation result in Fig. 2. The current flowing to the left loop and the right loop are 0.500826A and 0.499174 A and they match very well.



Without resistive sheet　　With resistive sheet

Fig. 2. The current comparison in the conductor.

The second application is used to examine the effectiveness of the coaxial conductors support. In this project, there are three coaxial conductors. The value and the direction of the current in each conductor are shown in Fig. 3.

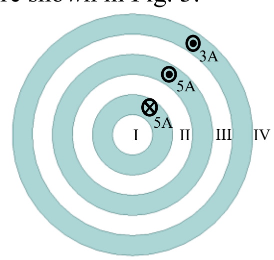


Fig. 3. The value and the direction of the current in the conductors

From physical point of view, the H field in the air regions I and III should be zero, and H field in air regions II and IV should be nonzero. The result in Fig. 4 shows that the simulation is able to correctly model the physics.

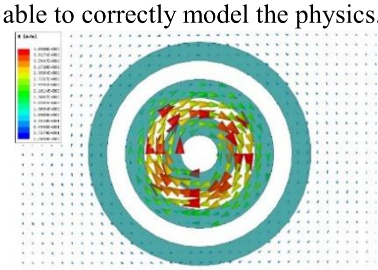


Fig. 4. The H field in computational region

Reference

[1] Henrotte F. and Hameyer K., "An algorithm to construct the discrete cohomology basis functions required for magnetic scalar potential formulations without cuts", *IEEE Trans. Magnetics*, 39(3):1167-1170, 2003.

[2] Ren Z., Zhou P. and Cendes Z.J., "Computation of current vector potential due to excitations in multiply connected conductors," in *Proc. ICEF Int. Conf.*, Tianjin, China, 2000, pp. 121-124.

[3] Ren Z., "***T***-Ω formulation for eddy-current problems in multiply connected regions", *IEEE Trans. Magnetics*, 38(2):557-560, 2002.

[4] P. Zhou, Z. Badics, D. Lin and Z. J. Cendes, "Nonlinear T-Ω formulation including motion for multiply connected 3-D problems," *IEEE Trans. Magnetics.*, vol. 44, Issue 6, pp. 718-721, May. 2008.

[5] C. Lu, P. Zhou, D. Lin, B. He, D. Sun, "Multiply connected 3-D transient problem with rigid motion associated with T-Ω formulation," *IEEE Trans. Magnetics.*, vol. 50, Issue 2, pp. 449-452, Feb. 2014.